HYPER VERSION

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From p-branes to fluxbranes and back

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Abstract: In this note we study aspects of the interplay between fluxbranes and p-branes. We describe how a fluxbrane can be physically realized as a limit of a brane-antibrane configuration, in a manner similar to the way a uniform electric field appears in between the plates of a capacitor. We also study the evolution of a fluxbrane after nucleation of p-branes. We find that Kaluza-Klein fluxbranes do relax by forming brane-antibrane pairs or spherical branes, but we also find that for fluxtubes with dilaton coupling in a different range, the field strength does not relax, instead it becomes stronger after each nucleation bounce. We speculate on a possible runaway instability of such fluxtubes and an eventual breakdown of their classical description.

Keywords: p-branes, D-branes, Black Holes in String Theory, Black Holes.

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1. Introduction

In recent times two classes of localized solutions of General Relativity and of the supergravity theories that derive from string/M-theory have become the subject of a detailed study. The most prominent one consists of black holes (in a broad sense which includes their possibly singular charged extremal limits) and their higher dimensional counterparts, p-branes. Another class consists of self-gravitating bundles of lines of flux, generically known as fluxbranes. The oldest known fluxbrane is a solution in Einstein-Maxwell theory known as the Melvin universe [1]. It describes a fluxtube where a finite amount of flux is confined by its own self gravity. This solution can be generalized [2, 3] and in recent months such fluxbranes have found considerable attention in string theory [4]–[21].

Since p-branes are typically sources for gauge fields, while fluxbranes consist of force lines of these gauge fields, it is clear that there must be interesting dynamics arising from their interactions. In this note we analyze two aspects of the interplay between them. We will exhibit how fluxbranes can arise from brane-antibrane configurations. One may then consider p-branes as the primary objects of the theory, and our result gives a physical realization of a fluxbrane in terms of them — fluxbranes from p-branes. On the other hand, a typical fluxbrane is unstable to spontaneous formation of spherical branes or brane-antibrane pairs — p-branes from fluxbranes. This can be seen as the mechanism by which a fluxbrane relaxes. We aim to study how, and even if, this relaxation takes place.

While it is possible to consider fluxbrane configurations involving more than one gauge field [22, 23], for simplicity we will only study the case of a single U(1) field. Consider the four dimensional Einstein-Maxwell-Dilaton system with an arbitrary dilaton coupling a,

$$S = \frac{1}{16\kappa}\int d^4x\sqrt{-g}\left(R - 2\partial_\mu\phi\partial^\mu\phi - e^{2a\phi}F^2\right). \tag{1.1}$$

For $a = 0$ the scalar ϕ decouples and we have the original Einstein-Maxwell theory, while for $a = \sqrt{3}$ the action can be derived from the five dimensional Einstein action via Kaluza-Klein reduction. The dilatonic Melvin fluxtube solution for an arbitrary a takes the form [3]

$$\begin{aligned} ds^2 &= \Lambda^{\frac{2}{1+a^2}}(-dt^2 + dz^2 + d\rho^2) + \Lambda^{-\frac{2}{1+a^2}}\rho^2 d\varphi^2 \,, \\ e^{2a(\phi-\phi_0)} &= \Lambda^{\frac{2a^2}{1+a^2}} \,, \qquad A_\varphi = e^{-a\phi_0}\frac{b\rho^2}{2\Lambda} \,, \end{aligned} \tag{1.2}$$

where

$$\Lambda = 1 + \frac{1+a^2}{4}b^2\rho^2 \,. \tag{1.3}$$

The solution is parameterized by ϕ_0, the value of the dilaton at the center of the fluxtube, and b, which characterizes the strength of the magnetic field at the origin. A peculiar feature of the Melvin solution is that the total integrated

$$Flux = \oint_{S_\infty} A_\varphi = e^{-a\phi_0}\frac{4\pi}{(1+a^2)}\frac{1}{b} \,, \tag{1.4}$$

is finite and inversely proportional to b. Hence, in the limit $b \to 0$, even if the fieldstrength goes to zero at the center and the metric becomes flatter, the total flux diverges.

We will restrict ourselves to fluxbranes of codimension two. Fluxbranes of higher codimension have been studied in [4, 5, 7], but the exact analytic solutions are not fully known.

2. Fluxbranes from brane-antibrane pairs

An approximately uniform electric field is physically realized as the field in between, and away from the edges of, the plates of a large capacitor. A uniform field extending throughout all of space then results as the limit where the size of the plates, as well as the distance between them, grows to infinity. Here we show that, similarly, a fluxbrane can be obtained as a limiting case of the field in between a brane-antibrane pair.

Brane-antibrane configurations of the sort we need are explicitly known only for the case of branes of codimension three. In four dimensions, these solutions describe a static pair of black holes of opposite charges; in [24] they were dubbed 'diholes,' see also [25, 26, 27]. The two oppositely charged black holes can be submerged in a Melvin fluxbrane background, which can be tuned so that the attraction between the black holes is balanced. However we will also consider the general situation where the field is not tuned to the equilibrium value. Generically, then, conical singularities (cosmic strings) will be present.

The solution for the dihole in a Melvin fluxtube can be expressed in several different coordinates. In Boyer-Lindquist-like coordinates [24, 28, 29], the metric is given by

$$ds^2 = \Lambda^{\frac{2}{1+a^2}}\left(-dt^2 + \frac{\Sigma^{\frac{4}{1+a^2}}}{(\Delta + (m^2+k^2)\sin^2\theta)^{\frac{3-a^2}{1+a^2}}}\left(\frac{dr^2}{\Delta} + d\theta^2\right)\right) + \frac{\Delta\sin^2\theta}{\Lambda^{\frac{2}{1+a^2}}}d\varphi^2 \,, \tag{2.1}$$

the gauge field,

$$A_\varphi = -e^{-a\phi_0}\frac{\frac{2}{\sqrt{1+a^2}}mrk + \frac{1}{2}b((r^2-k^2)^2 + \Delta k^2\sin^2\theta)}{\Lambda\Sigma}\sin^2\theta \,, \tag{2.2}$$

and the dilaton

$$e^{\phi-\phi_0} = \Lambda^{\frac{a}{1+a^2}}\,, \tag{2.3}$$

where Λ is

$$\Lambda = \frac{\Delta + k^2\sin^2\theta + 2\sqrt{1+a^2}bmrk\sin^2\theta + \frac{1+a^2}{4}b^2\sin^2\theta((r^2-k^2)^2 + \Delta k^2\sin^2\theta)}{\Sigma}\,. \tag{2.4}$$

Here we have also defined

$$\begin{aligned} \Delta &= r^2 - 2mr - k^2\,, \\ \Sigma &= r^2 - k^2\cos^2\theta\,. \end{aligned} \tag{2.5}$$

The parameter m can be viewed as determining the mass and charge of each of the black holes, while k is related to the distance between them.[1] On the other hand, b is related to the presence of a magnetic field and it is easy to see that as we take $r \to \infty$ the solution (2.1)–(2.3) asymptotes to the dilatonic Melvin solution (1.2) with magnetic field parameter b and dilaton expectation value ϕ_0.

Note that the Killing vector ∂_φ for the axial symmetry has vanishing norm at $\theta = 0, \pi$ as well as at $r = r_+ = m + \sqrt{m^2+k^2}$. The proper interpretation of this fact is that the symmetry axis has two parts: an 'outer' part, where $\theta = 0, \pi$ with $r_+ \le r < \infty$, and an 'inner' part, where $r = r_+$ and $0 \le \theta \le \pi$. The location where the two segments intersect, $r = r_+$, $\theta = 0, \pi$, corresponds to the location of two oppositely charged extremal dilatonic black holes; in particular, for $a = 0$ one finds oppositely charged, extremal Reissner-Nordstrom black holes. In general such a configuration can not be static, since the black holes attract. This fact manifests itself in the presence of deficit angles on the outer and inner axis

$$\delta_{\mathrm{outer}} = 2\pi - \Delta\varphi\,, \qquad \delta_{\mathrm{inner}} = 2\pi - \left(1+\frac{m^2}{k^2}\right)^{\frac{2}{1+a^2}}\left(1+\frac{\sqrt{1+a^2}bmr_+}{k}\right)^{-\frac{4}{1+a^2}}\Delta\varphi, \tag{2.6}$$

where $\Delta\varphi$ is the angular periodicity $\varphi \sim \varphi + \Delta\varphi$. These deficit angles represent 'strings' and 'struts' which push apart or pull together the black holes.

If we want to eliminate the conical deficits both at the inner and outer axis, we can do so by choosing the periodicity $\Delta\varphi = 2\pi$ and the asymptotic magnetic field $b = k/(\sqrt{1+a^2}mr_+)(-1+\sqrt{1+m^2/k^2})$. This configuration represents a pair of oppositely charged black holes held in an (unstable) equilibrium by an external Melvin magnetic flux-tube. In other situations, when the field is not tuned to the equilibrium value, we will choose $\Delta\varphi$ so as to have $\delta_{\mathrm{inner}} = 0$, $\delta_{\mathrm{outer}} > 0$.

We want to identify possible realizations of Melvin fluxtubes as different limits of the solution above. To this effect, it is convenient to transform the dihole metric into cylindrical Weyl coordinates, so r, θ are replaced by z, ρ. Defining

$$R_\pm = \sqrt{\rho^2 + (z \pm \sqrt{m^2+k^2})^2}\,, \tag{2.7}$$

[1] See [24] for a detailed analysis.

the change of variables is accomplished by (see [27] for details)

$$r - m \equiv R = \frac{1}{2}(R_+ + R_-)\,, \qquad \cos\theta = \frac{z}{R} = \frac{R_+ - R_-}{2\sqrt{m^2+k^2}}\,, \qquad \rho = \sqrt{\Delta}\sin\theta. \quad (2.8)$$

Let us first focus on the case of $b = 0$, where the brane and antibrane are kept apart by cosmic strings without any external fluxtube. In order to have the axis $r = r_+$ free of conical singularities, we define

$$\tilde{\varphi} = \left(1 + \frac{m^2}{k^2}\right)^{\frac{2}{1+a^2}} \varphi \quad (2.9)$$

and then identify $\tilde{\varphi} \sim \tilde{\varphi} + 2\pi$.

We want the size of the black holes, as well as their separation, to grow very large, in a manner that the magnetic field stretching between them remains finite. This is a limit where both m and k grow to ∞. To achieve this, scale

$$m \to \lambda^{\frac{3+a^2}{1+a^2}} m\,, \qquad k \to \lambda^{\frac{2}{1+a^2}} k \quad (2.10)$$

and $\rho \to \lambda^{\frac{2}{1+a^2}}\rho$, $z \to \lambda^{\frac{2}{1+a^2}} z$, $t \to \lambda^{\frac{2}{1+a^2}} t$. Note we do not scale $\tilde{\varphi}$, so its periodicity remains unaltered. Take now $\lambda \to \infty$. In this limit the conical deficit for the strings outside the dihole becomes equal to 2π (i.e., the geometry collapses), but since we stay in between the branes, this singularity is pushed away to infinity. The metric becomes

$$\begin{aligned} ds^2 = & \left(1 + \frac{\rho^2}{k^2}\right)^{\frac{2}{1+a^2}} \left[-\left(\frac{k}{2m}\right)^{\frac{4}{1+a^2}} dt^2 + \left(\frac{2k}{m}\right)^{\frac{4}{1+a^2}} (d\rho^2 + dz^2)\right] + \\ & + \left(\frac{2k}{m}\right)^{\frac{4}{1+a^2}} \frac{\rho^2}{(1+\rho^2/k^2)^{\frac{2}{1+a^2}}} d\tilde{\varphi}^2\,, \end{aligned} \quad (2.11)$$

the magnetic potential,

$$A_{\tilde{\varphi}} = e^{-a\phi_0} \lambda^{\frac{2a^2}{1+a^2}} \left(\frac{k}{m}\right)^{\frac{3-a^2}{1+a^2}} \frac{4m}{1+\rho^2/k^2}\,, \quad (2.12)$$

and the dilaton,

$$\phi - \phi_0 = \frac{1}{1+a^2}\log\left(1 + \frac{\rho^2}{k^2}\right) + \frac{2a}{1+a^2}\log\left(\frac{2\lambda m}{k}\right). \quad (2.13)$$

Both $A_{\tilde{\varphi}}$ and ϕ diverge as $\lambda \to \infty$. However, the divergence is a constant (independent of the coordinates) that can be absorbed by shifting the dilaton,

$$\phi_0 \to \phi_0 - \frac{2a}{1+a^2}\log\left(\frac{2\lambda m}{k}\right). \quad (2.14)$$

Then, after further rescaling $\bar{t} = \left(\frac{k}{2m}\right)^{\frac{2}{1+a^2}} t$, $\bar{z} = \left(\frac{2k}{m}\right)^{\frac{2}{1+a^2}} z$, $\bar{\rho} = \left(\frac{2k}{m}\right)^{\frac{2}{1+a^2}} \rho$ one recovers the dilatonic Melvin fluxtube (1.2), with

$$b = \frac{2}{\sqrt{1+a^2}} \frac{1}{k} \left(\frac{m}{2k}\right)^{\frac{2}{1+a^2}}\,. \quad (2.15)$$

Hence we have found a physical realization of a fluxtube as a limit of the field created by a brane-antibrane pair.

Now we study the limit where two black holes in an external fluxtube are moved apart, $k \to \infty$, but m, and hence their size, is kept finite. One might think that in this case one should recover the same field b as in the absence of the black holes. However, it is possible to take the limit in such a way that the field left over after the black holes have been removed actually differs from the initial one. To do so, after taking the limit $k \to \infty$, rescale t, ρ and z by a factor of $(1+\sqrt{1+a^2 bm})^{2/(1+a^2)}$ and choose the periodicity

$$\Delta\varphi = 2\pi \left(1+\sqrt{1+a^2}\, bm\right)^{\frac{4}{1+a^2}}. \tag{2.16}$$

It follows from (2.6) that there is a nonzero deficit angle on the outer axis, corresponding to a cosmic string pulling the black holes apart. However, in the limit $k \to \infty$ the outer segments are pushed to infinity and only the inner axis with zero deficit angle remains. The two black holes forming the dihole have been pulled to infinity and a Melvin spacetime remains, with parameters (1.2), with the parameters $\hat{b}, \hat{\phi}_0$ replacing b, ϕ_0:

$$\hat{b} = \frac{b}{(1+\sqrt{1+a^2 bm})^3}, \qquad e^{\hat{\phi}_0} = e^{\phi_0}\left(1+\sqrt{1+a^2}\, bm\right)^{\frac{a}{1+a^2}} \tag{2.17}$$

and the total integrated flux is given by

$$\text{Flux} = \oint_{S_\infty} A_\phi = e^{-a\hat{\phi}_0} \frac{4\pi}{1+a^2} \frac{1}{\hat{b}}. \tag{2.18}$$

Note that $(1+\sqrt{1+a^2 bm}) > 1$ and hence the new coupling constant is bigger. The magnetic field strength at the center of the fluxtube is smaller. This shows that moving the diholes away has made the magnetic field weaker. It is one of the curious features of the Melvin solution that the total flux is getting larger. This is caused by the fact that the magnetic flux, though weaker, can spread out further in a flatter spacetime such that the total flux is increased.

3. Fluxtube evolution through pair production bounces

Generically, fluxbranes are unstable. Except for the cases where they are supersymmetric, they can nucleate brane-antibrane pairs, or spherical branes, in a manner analogous to Schwinger pair production. One expects that fluxbranes relax via this process.

The production of black hole pairs in the presence of a fluxtube can be described semi-classically by a gravitational instanton bounce. This is obtained by analytic continuation [2, 30, 31, 32] of the Ernst solution [33], which describes a pair of black holes accelerating apart in an external magnetic field. The dilatonic generalization of these solutions was found in [31, 32].

The Ernst solution is known to asymptote, at large spatial distances, to a Melvin fluxtube. This is the field on the axis outside the black holes, and is naturally regarded as the fluxtube that nucleates the pair. We will show below that the Ernst solution also approaches a Melvin fluxtube at *future* asymptotic infinity.[2] This fluxtube is the field left

[2]By time reversal invariance, the same field is present at past infinity. However, the latter is not relevant when considering a pair creation process.

over between the black holes when they have got infinitely apart, and can therefore be viewed as the fluxtube that remains after pair production. In this way we can follow the evolution of the fluxtube after successive bounces.

In the following we use the results and notation of [31, 32]. The dilatonic version of the Ernst metric is

$$ds^2 = \frac{1}{A^2}\frac{1}{(x-y)^2}\left\{\Lambda^{\frac{2}{1+a^2}}\left(F(x)G(y)dt^2 - \frac{F(x)}{G(y)}dy^2 + \frac{F(y)}{G(x)}dx^2\right) + \Lambda^{-\frac{2}{1+a^2}}F(y)G(x)d\varphi^2\right\}. \tag{3.1}$$

The dilaton and gauge field are given by

$$e^{2a(\phi-\phi_0)} = \Lambda^{\frac{2a^2}{1+a^2}}\frac{F(y)}{F(x)}, \qquad A_\varphi = -\frac{2e^{-a\phi_0}}{(1+a^2)b\Lambda}\left(1+\frac{1+a^2}{2}bqx\right), \tag{3.2}$$

where

$$\Lambda = \left(1+\frac{1+a^2}{2}bqx\right)^2 + \frac{(1+a^2)b^2}{4A^2(x-y)^2}G(x)F(x). \tag{3.3}$$

The functions $F(\xi)$ and $G(\xi)$ are defined by

$$\begin{aligned} F(\xi) &= (1+r_-A\xi)^{\frac{2a^2}{1+a^2}} \\ &= (r_-A)^{\frac{2a^2}{1+a^2}}(\xi-\xi_1)^{\frac{2a^2}{1+a^2}} \\ G(\xi) &= (1+\xi^2-r_+A\xi^3)(1+r_-A\xi)^{\frac{1-a^2}{1+a^2}} \\ &= -(r_+A)(r_-A)^{\frac{1-a^2}{1+a^2}}(\xi-\xi_1)^{\frac{1-a^2}{1+a^2}}(\xi-\xi_2)(\xi-\xi_3)(\xi-\xi_4) \end{aligned} \tag{3.4}$$

and $q = \sqrt{r_+r_-/(1+a^2)}$.

The zeros of the fourth order polynomial $G(\xi)$ in (3.4) are denoted $\xi_1 \le \xi_2 \le \xi_3 \le \xi_4$. A fixed minkowskian signature is enforced by having $y < x$. The surface $y = \xi_2$ is a black hole horizon and $y = \xi_3$ is an acceleration horizon. Note also that $\xi_1 = \xi_2$ corresponds to the limit of extremal black holes. The variable x is a polar angle and lies in $\xi_3 \le x \le \xi_4$. The absence of conical singularities at the poles $x = \xi_3, \xi_4$ requires firstly

$$G'(\xi_3)\Lambda(\xi_4)^{\frac{2}{1+a^2}} = -G'(\xi_4)\Lambda(\xi_3)^{\frac{2}{1+a^2}}, \tag{3.5}$$

and then fixing the period of φ to

$$\Delta\varphi = \frac{4\pi(\Lambda(\xi_3))^{\frac{2}{1+a^2}}}{G'(\xi_3)}. \tag{3.6}$$

The four parameters of the solution (3.1)–(3.3), A, b, r_+ and r_-, correspond, roughly,[3] to the acceleration, magnetic fluxtube field, outer and inner horizons of the black hole, respectively. Imposition of (3.5) leaves only three of these as independent, by essentially enforcing Newton's law $mA \approx qb$ (with $m = (r_+ + r_-)/2$). If the solution is continued to euclidean time, so as to provide an instanton for pair production, a further restriction need be imposed, namely, the equality of the temperatures of the black hole and acceleration horizons. This requires the black holes to be extremal or nearly extremal, $m \approx q$.

[3]The correspondence is accurate only in the limit of small black holes, $r_+, r_- \ll 1/A, 1/b$.

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As explained in [31], the Ernst solution becomes a Melvin background for large space-like distances. This limit corresponds to $x \to \xi_3, y \to \xi_3$. The resulting Melvin spacetime has parameters

$$\tilde{b} = \frac{bG'(\xi_3)}{2(\Lambda(\xi_3))^{\frac{3+a^2}{2(1+a^2)}}}, \qquad e^{\tilde{\phi}_0} = e^{\phi_0}\Lambda(\xi_3)^{\frac{a}{1+a^2}}\,. \tag{3.7}$$

This is the field present *before* the black holes are nucleated.

We are interested in the 'leftover' spacetime after the black holes have accelerated away to infinity. To do so, we shall go to future asymptotic infinity while remaining between the two black holes. The coordinates in (3.1) only cover the part of the spacetime containing one black hole, up to $y = \xi_3$. The latter is an acceleration horizon, and the coordinate patch can be continued beyond it. For $\xi_3 < y < \xi_4$, the coordinate y is timelike and t is spacelike. We can access all this region, while keeping $y < x$, by staying at the portion of the axis that runs between the black holes, $x = \xi_4$. In this region (the 'upper wedge'), the geometry is not static: it evolves from the point of closest approach between the black holes, at $y = \xi_3$, to the limit where they are infinitely far apart, at $y = \xi_4$.

To obtain the form of the solution at asymptotically late times, we change variables as

$$x = \xi_4 - \frac{\rho^2}{T^4}\,, \qquad y = \xi_4 - \frac{1}{T^2}\,, \qquad t = \frac{z}{T}\,. \tag{3.8}$$

Taking the limit $T \to \infty$, one gets

$$\begin{aligned}
\Lambda(x,y) &= \Lambda(\xi_4) - \frac{(1+a^2)b^2 F(\xi_4)G'(\xi_4)}{4A^2}\rho^2 + o(1/T) \\
A_\varphi &= -\frac{2e^{a\phi_0}}{(1+a^2)b\Lambda}\Lambda(\xi_4)^{\frac{1}{2}} + o(1/T) \\
ds^2 &= \frac{F(\xi_4)}{A^2}\left\{\Lambda^{\frac{2}{1+a^2}}\left(-G'(\xi_4)dz^2 + \frac{4}{G'(\xi_4)}dT^2 - \frac{4}{G'(\xi_4)}d\rho^2\right) - \Lambda^{-\frac{2}{1+a^2}}G'(\xi_4)\rho^2 d\varphi^2\right\} + \\
&\quad + o(1/T)\,.
\end{aligned} \tag{3.9}$$

After a further rescaling

$$\begin{aligned}
\hat{\rho} &= \frac{2\Lambda(\xi_4)^{\frac{1}{1+a^2}}}{A}\sqrt{\frac{F(\xi_4)}{|G'(\xi_4)|}}\rho\,, \\
\hat{T} &= \frac{2\Lambda(\xi_4)^{\frac{1}{1+a^2}}}{A}\sqrt{\frac{F(\xi_4)}{|G'(\xi_4)|}}T\,, \\
\hat{z} &= \frac{\Lambda(\xi_4)^{\frac{1}{1+a^2}}}{A}\sqrt{F(\xi_4)|G'(\xi_4)|},
\end{aligned} \tag{3.10}$$

the $T \to \infty$ limit is a static Melvin spacetime. We denote the new parameters with hats,

$$\hat{b} = \frac{b|G'(\xi_4|)}{2(\Lambda(\xi_4))^{\frac{3+a^2}{2(1+a^2)}}}\,, \qquad e^{\hat{\phi}_0} = e^{\phi_0}(\Lambda(\xi_4))^{\frac{a}{1+a^2}}\,. \tag{3.11}$$

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We can now compare the Melvin background for large spatial distances (3.7) with the Melvin background after the black holes have accelerated away (3.11). Using (3.5) one finds

$$\frac{\hat{b}^2}{\tilde{b}^2} = \left(\frac{\Lambda(\xi_3)}{\Lambda(\xi_4)}\right)^{\frac{a^2-1}{a^2+1}}, \qquad \frac{e^{\hat{\phi}_0}}{e^{\tilde{\phi}_0}} = \left(\frac{\Lambda(\xi_4)}{\Lambda(\xi_3)}\right)^{\frac{a}{1+a^2}}. \tag{3.12}$$

Since $\xi_4 > \xi_3$ it follows from (3.3) that $\Lambda(\xi_4) > \Lambda(\xi_3)$, so the 'leftover' coupling is always larger than the asymptotic coupling. This also implies that, for $a > 1$ the 'leftover' field strength decreases, $\hat{b} < \tilde{b}$. Hence, these fluxbranes do relax after a bounce.

However, for $a < 1$ the leftover fieldstrength *increases*, $\hat{b} > \tilde{b}$. Contrary to expectation, the strength of these fluxtubes appears to build up after pair creation. The fluxtube concentrates more with each bounce. This has a striking consequence. The rate for pair creation is approximately given, for small black holes (which are the ones more likely to be produced) by

$$\Gamma \sim e^{-\frac{\pi m^2}{q\tilde{b}}}, \tag{3.13}$$

hence it is larger for larger initial field $\tilde{b}$. Since $\tilde{b}$ is enhanced after each bounce, pair production becomes increasingly likely and the fluxtube starts a runaway process, by creating black hole pairs at an ever increasing rate.

Notice that this instability is not incompatible with the instanton action being positive, hence yielding a suppressed rate for each bounce. The latter implies that the build-up of the fluxtube strength will proceed slowly. It is not incompatible either with energy conservation, since the total energy is conserved in the pair production process [34]. The total energy of a fluxtube that extends to infinity is infinite, so it will keep producing black hole pairs, and increasing its field strength, until it reaches Planck-size values and its semi-classical description ceases to be valid. For a physical fluxtube of finite extent, one should take into account the fact that its energy must decrease with each pair that is produced. In a sense, this instability is reminiscent of Hawking evaporation, for which the emission rate for a neutral black hole increases as the black hole evaporates.

It is interesting that this property depends on the value of the dilaton coupling a. This is likely correlated with the fact that the nature of the φ coordinate depends on the value of a. For $a < 1$ the circle size shrinks to zero as $\rho \to \infty$ and the space closes off, whereas for $a > 1$ it grows.

4. Higher Kaluza-Klein fluxbranes

It is an interesting fact that for Kaluza-Klein theories the Melvin spacetime can be obtained as a compactification of flat space on a circle with twisted identifications [35, 36]. Starting with d dimensional flat space

$$ds^2 = -dt^2 + dx_1^2 + \cdots + dx_{d-4}^2 + dr^2 + r^2 d\varphi^2 + R^2 dy^2, \tag{4.1}$$

where y is periodic with period 2π, one reduces along the orbits of the Killing vector $\partial_y + q\partial_\varphi$, which means that a translation $y \to y + 2\pi$ is accompanied by a rotation $\varphi \to$

$\varphi + 2\pi qR$. It is useful to introduce a new single valued angular variable $\tilde{\varphi} = \varphi - qRy$ which has standard periodicity. In these coordinates the metric becomes

$$ds^2 = -dt^2 + dx_1^2 + \cdots + dx_{d-4}^2 + dr^2 + r^2(d\tilde{\varphi} + qRdy)^2 + R^2 dy^2 \,. \tag{4.2}$$

Using the standard formulae for Kaluza-Klein reduction,

$$ds_d^2 = e^{\frac{4}{\sqrt{d-2}}\phi}(dy + 2A_\mu dx^\mu)^2 + e^{-\frac{4}{(d-3)\sqrt{d-2}}\phi} ds_{d-1}^2 \,, \tag{4.3}$$

rescaling can bring the metric into the following canonical form

$$\begin{aligned} ds_{d-1}^2 &= (1+\tilde{q}^2 r^2)^{\frac{1}{d-3}} \left(-dt^2 + dx_1^2 + \cdots + dx_{d-4}^2 + dr^2 + \frac{r^2}{1+\tilde{q}^2 r^2} d\tilde{\varphi}^2 \right) \\ e^{\frac{4}{\sqrt{d-2}}\phi} &= R^2(1+\tilde{q}^2 r^2) \,, \qquad A_{\tilde{\varphi}} = \frac{\tilde{q} r^2}{2R^{\frac{d-2}{d-3}}(1+\tilde{q}^2 r^2)} \,, \qquad \tilde{q} = \frac{q}{R^{\frac{1}{d-3}}} \,. \end{aligned} \tag{4.4}$$

The total flux is given by $\oint A_{\tilde{\varphi}} = \pi/(qR)$. Such a solution can be called a flux $(d-4)$-brane, since it enjoys $(d-3)$ dimensional Poincare invariance. Setting $d = 11$ produces a RR-flux seven brane in type-IIA string theory. For $q = 1/R$ the twist is a rotation by 2π which acts as $(-1)^F$ on fermions, which means that fermions have anti-periodic boundary conditions along the M-theory circle. Such a M-theory background is believed to be type-0A theory [37]. This fact lead to the conjecture [38] that a Melvin flux seven brane with $q = 1/R$ is a dual description of type 0A.

In dimensions higher than four, there are several ways to generalize a black hole-anti-black hole pair. One is to add flat dimensions, resulting in planar, codimension three brane-antibrane pairs. These are infinite in extent. But one can also have a configuration of finite size in the form of spherical branes. Static, S^p-spherical Kaluza-Klein branes in $p+4$ dimensions were constructed in [36]. They can be held in equilibrium by 'conical branes' or by Melvin fluxbranes. In the case of a spherical p-brane without any external fluxbranes, it is easy to identify a limit where the p-brane grows very large in size, while the geometry and fields in its interior approach a Kaluza-Klein Melvin fluxbrane. This is analogous to the way in which we obtained a fluxtube from a dihole in (2.11).

In a similar way, it is possible to construct a solution with a pair of planar, infinite brane and antibrane accelerating away, but, unless the planar directions are compactified, the euclidean action of these solutions is infinite and therefore gives a vanishing decay rate. More appropriately, the process of pair creation in the presence of a higher dimensional Melvin fluxbranes is replaced by the creation of spherical KK-branes (for $d = 11$ these are D6-branes). These nucleate via an instanton of finite action and exponentially expand after their nucleation [36]. In the following we investigate the minkowskian evolution after the creation of the brane and look for the 'leftover' spacetime.

The gravitational instanton mediating the creation of KK-branes in a Melvin background is given by the euclidean Myers-Perry [39] black hole (see [36]),

$$\begin{aligned} ds^2 &= \left(1 - \frac{m}{r^{d-5}\Sigma}\right) dx_d^2 - \frac{2mk\sin^2\theta}{r^{d-5}\Sigma} dx_d d\varphi + \frac{\Sigma}{r^2 - k^2 - mr^{5-d}} dr^2 + \Sigma d\theta^2 + \\ &+ \frac{\sin^2\theta}{\Sigma}\left((r^2 - k^2)\Sigma - \frac{m}{r^{d-5}} k^2 \sin^2\theta\right) d\varphi^2 + r^2\cos^2\theta d\Omega_{d-4} \,, \end{aligned} \tag{4.5}$$

where, again, $\Sigma = r^2 - k^2\cos^2\theta$. The minkowskian horizon is rotated to a euclidean 'bolt', with radius r_h defined by

$$r_+^2 - k^2 - \frac{m}{r_+^{d-5}} = 0\,. \tag{4.6}$$

The absence of a conical singularity at $r = r_+$ then determines the radius R of the Kaluza-Klein direction x_d. The second quantity characterizing the black hole solution is the (analytically continued) angular momentum Ω. In terms of m and k, these are

$$R = \frac{2mr_+^{6-d}}{(d-3)r_+^2 - (d-5)k^2}\,, \qquad \Omega = \frac{kr_+^{d-5}}{m}\,. \tag{4.7}$$

Since (4.5) is asymptotically flat one can embed the black hole in a Melvin fluxbrane by twisting. However the identifications have to be globally well defined and this implies that there are two possible choices of twist angle q,

$$q_{0\mathrm{A}} = \Omega\,, \qquad q_{\mathrm{IIA}} = \Omega - \frac{sgn(\Omega)}{R}\,. \tag{4.8}$$

The two twists differ by a rotation of 2π, which changes the boundary conditions on spacetime fermions from antiperiodic to periodic. The reduced metric for either q can be expressed using the function

$$\Lambda = R^2\left(1 - \frac{m}{r^{d-5}\Sigma} - q\frac{2mk\sin^2\theta}{r^{d-5}\Sigma} + q^2\frac{\sin^2\theta}{\Sigma}\left((r^2-k^2)\Sigma - mr^{5-d}k^2\sin^2\theta\right)\right). \tag{4.9}$$

Then the dilaton and gauge field are

$$e^{\frac{4}{\sqrt{d-2}}\phi} = \Lambda\,, \qquad A_\varphi = \frac{R}{2\Lambda}\frac{\sin^2\theta}{\Sigma}\left(-\frac{mk}{r^{d-5}} + q\left((r^2-k^2)\Sigma - mr^{5-d}k^2\sin^2\theta\right)\right). \tag{4.10}$$

We are interested in the minkowskian evolution of the spacetime after the nucleation of a brane. The analytic continuation of one of the ignorable angular variables of the $d-4$ sphere results into a boost coordinate that then serves as the timelike coordinate after nucleation. The lorentzian metric post-nucleation is then given by

$$\begin{aligned} ds^2 &= \Lambda^{\frac{1}{d-3}}\left\{\frac{\Sigma}{r^2-k^2-mr^{5-d}}dr^2 + \Sigma d\theta^2 + r^2\cos^2\theta(-dt^2 + \cosh^2 t d\Omega_{d-5}^2) + \right.\\ &\quad \left. + \frac{R^2}{\Lambda}\sin^2\theta(r^2-k^2-mqr^{5-d})d\varphi^2\right\}. \end{aligned} \tag{4.11}$$

As explained in [36] this metric for the choice $q = q_{\mathrm{IIA}}$ describes a spherical D6-brane expanding in a Flux 7-brane background. It is natural to ask what the 'leftover' spacetime after the D6-brane has moved to infinity looks like. The metric (4.11) has an acceleration horizon and only covers the region of spacetime inside it. To continue past it, it is useful to make some coordinate changes. Firstly define $z = r\cos\theta$ and $\tilde{r} = f(r)\sin\theta$, where

$$\frac{1}{f}\frac{df}{dr} = \frac{r}{r^2-k^2-mr^{5-d}}\,. \tag{4.12}$$

Secondly, define Rindler like coordinates X, T in terms of z, t by $z = \sqrt{X^2 - T^2}$ and $t = \operatorname{arctanh}(T/X)$. The exact form of the metric in the new coordinates is very complicated, but we are only interested in the $T \to \infty$ limit. So we need simply analyze the leading part of the solution, dropping terms of order $1/T$.

Now, in order to get a static metric in terms of the new coordinates, the old radial coordinate has to behave as

$$r = r_+ + \left(\frac{\tilde{r}}{T}\right)^{1/c_h} = r_+ + \frac{1}{4c_h^2}\left(\frac{\hat{r}}{T}\right)^2, \tag{4.13}$$

where we have defined $c_h = Rr_+^{d-4}/(2\mu)$. Then, as $T \to \infty$, (4.9) becomes

$$\Lambda = R^2\left(\left(1 - \frac{q}{\Omega}\right)^2 + \frac{k^2}{r_+^3 c_h}\frac{q^2}{R^2\Omega^2}\hat{r}^2\right) + O(1/T), \tag{4.14}$$

with the gauge field

$$A_\varphi = \frac{R}{2\Lambda}\left(\frac{q-\Omega}{\Omega^2} + \frac{k^2}{r_+^3 c_h}\frac{q}{R^2\Omega^2}\hat{r}^2\right) + O(1/T), \tag{4.15}$$

and the metric

$$ds^2 = \Lambda^{\frac{1}{d-3}}\left\{-\frac{k^2}{r_+^2}dT^2 + \frac{k^2}{r_+^3 c_h}d\hat{r}^2 + dX^2 + X^2 d\Omega_{d-5}^2 + \frac{1}{\Omega^2\Lambda}\frac{k^2}{r_+^3 c_h}\hat{r}^2 d\varphi^2\right\} + O(1/T). \tag{4.16}$$

Note that for $q = q_{0\mathrm{A}}$ the constant term in (4.14) vanishes, which means that the metric is not of Melvin form. This is sensible, since the bounce for the 0A twist corresponds to an expanding 'bubble of nothing' [40].

For $q = q_{\mathrm{IIA}}$ the metric can be brought into Melvin form. After a further finite rescaling of coordinates $T \to \Omega^{\frac{1}{d-3}} r_+/k\ T$, $\hat{r} \to \Omega^{\frac{1}{d-3}} r_+^{3/2} c_h^{1/2}/k\ \hat{r}$, the metric (4.16) can again be brought into the canonical form (4.4) with parameters

$$\hat{q} = q\Omega^{\frac{1}{d-3}}, \qquad e^{\hat{\phi}_0} = \Omega^{-\frac{\sqrt{d-2}}{2}}. \tag{4.17}$$

These are the parameters of the asymptotic fluxtube at future infinity, which we take as the field left over after nucleation of a spherical brane. We can compare it to the (tilded) field before nucleation (i.e., the asymptotic field at spatial infinity),

$$\left(\frac{\hat{q}}{\tilde{q}}\right)^2 = (R\Omega)^{\frac{2}{d-3}}, \qquad \frac{e^{\hat{\phi}_0}}{e^{\tilde{\phi}_0}} = (R\Omega)^{-\frac{\sqrt{d-2}}{2}}. \tag{4.18}$$

Note that it follows from (4.7) that $\Omega R < 1$. Hence, after the nucleated KK-brane has accelerated away, the field strength at the center of the fluxbrane decreases, corresponding to the field being discharged. On the other hand, the dilaton, and hence the compactification radius, has increased.

As a check that the continuation of the coordinates past the acceleration horizon employed in this section is correct, observe that if we set $d = 5$ and compare with the results of section 3 for $a = \sqrt{3}$, then (3.12) agrees with (4.18) when $\Omega R = (\Lambda(\xi_3)/\Lambda(\xi_4))^{1/2}$, which is indeed the case (see [35]).

5. Discussion

In this note we have discussed several ways in which a more complicated spacetime involving p-branes becomes a fluxbrane by taking a certain limit or looking at the long time limit of nonstatic spacetimes. First we discussed how two black holes with opposite charges, in an otherwise empty space, can give rise to a fluxtube as the field in between them. We let the black holes grow very large and far apart, and in the limit obtained an exact dilatonic Melvin fluxtube. Alternatively, we may have viewed this limit as keeping the size of the black holes and their distance fixed, and then focusing on the region near the middle point in between the black holes, and at a small distance from the axis. This region, then, is well approximated by a fluxtube.

Secondly, we discussed the spacetime describing accelerating black holes in Melvin spacetimes and found the 'leftover' spacetime after the nucleated black holes or branes have accelerated away to infinity. For dilaton coupling $a > 1$ the fieldstrength decreases after pair production, which we take as evidence that the field discharges. However, we have found a striking phenomenon for dilaton coupling $a < 1$. For these cases, the strength of the magnetic field *grows* with each pair that is produced.

We pointed out that this signals a runaway instability, which is present for the particular case of Einstein-Maxwell theory. One might then worry that the magnetic fields in our Universe could be dangerously unstable to exploding into a myriad black holes. However, for magnetic fields well below the Planck scale the exponential suppression yields an extremely slow rate for pair production, so the rate at which the field builds up is presumably too small to have any observable consequences.[4]

In this note we only considered nucleation processes 'on the axis,' which means that the black holes or branes are nucleated at the center of the fluxbrane. Nucleation can also happen off the axis (possibly with lower rate, since the field strength decreases as one moves away from the center). We expect that the qualitative features of such processes are similar to the ones discussed in this note. Note however that no exact solution generalizing the Ernst solution in this respect is known at present.

Also, we have only studied fluxbranes of codimension two. It appears reasonable to expect that there exist realizations of fluxbranes of higher codimensions as limits of brane-antibrane configurations similar to those studied in this paper. However, the required solutions are not known. It would be very interesting to determine which of these fluxbranes relax or not by spontaneous nucleation of p-branes.

Finally, we note that the case of the 11 dimensional KK-Melvin is especially interesting due to the conjectured relation of a 'critical' Melvin with $q_{\rm IIA} = \pm 1/R$ to ten dimensional 0A theory [38]. In this case the nucleation of D6 branes indeed relaxes the field and it is suggestive that such a process could be interpreted as dual to the perturbative tachyon condensation in type 0A.

[4]For example, for the magnetic fields at the surface of a neutron star, the average time for producing a single black hole pair is much larger than the age of the Universe.

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